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Method for Calculating Wing Loading During Maneuvering Flight along a Three-Dimensional Curved Path

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The aerodynamic forces due to the nonsteady motion of a lifting surface along a three-dimensional path are analyzed. For this purpose, available steady-state calculation methods, based on potential theory, were modified by retaining the nonsteady terms in the governing equations. The surface motion is limited to such cases where the linear analysis can be applied. As an example, the time-dependent forces and moments of a slender wing in various nonsteady motions were studied.

Introduction

METHODS such as vortex lattice or doublet lattice are widely used for calculating the pressure distribution about lifting surfaces in steady and oscillatory potential flows along a straight flight path. These methods can be modified further to solve the time-dependent lift variations on the wings of a maneuvering aircraft. While extending the steady-state lifting surface model for the calculation of nonsteady flight along a curved path, the following corrections have to be made:

1) Correction of downwash distribution on wing surface via a transformation which enables the statement and linearization of the nonsteady boundary condition of zero-flow assumption across the wing surface.

2) A modification in the pressure distribution term must be made, since the Bernoulli equation includes additional nonsteady effects.

3) A nonsteady wake model must be constructed, which releases vortex ring elements from the trailing edge as the planform circulation varies. The strength of each element must satisfy Kelvin's theorem of no net circulation generation. An additional improvement that is strongly recommended is the calculation of wake distortion in the wing-wake-induced velocity field. This calculation can provide valuable information about vortex wake shape and location behind the aircraft.

Modifications 1 and 2 are carrried out by solving the problem in a noninertial frame of reference that follows the wing in such a way that its x direction is always tangent to the flight path. The orthogonal displacement in this system is limited to small displacements. However, the path curvature should be such that the flow disturbances caused by the wing should remain small.

Following Katz and Weihs,[1] the transformed continuity equation should remain the same when stated in any Cartesian frame of reference; therefore, in the wing-following frame, it is:

$$\nabla^2\Phi = 0 \tag{1}$$

where $\Phi=\Phi\ (x,y,z)$ is the velocity potential defined in an inertial frame of reference only, and the x,y,z system serves in defining the values relative to the wing. Therefore, the derivatives $\partial\Phi/\partial x$, $\partial\Phi/\partial y$, and $\partial\Phi/\partial z$ are the velocities parallel to the x,y,z directions, respectively, as measured in an inertial (x^*,y^*,z^*) coordinate system.

The transformed boundary conditions are:

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=0} = \left[U+\cancelto{\sim 0}{\frac{\partial\Phi}{\partial x}}-ry\right]\frac{\partial h}{\partial x}-qx+py+\frac{\partial h}{\partial t} \tag{2}$$

where $U=U(t)$ and $z=h(x,y,t)$ are the momentary flight velocity and orthogonal wing displacement, respectively, while (p,q,r) are the angular velocity components and

$$\lim_{|x|y|z|\to\infty}\nabla\Phi = 0 \tag{3}$$

The corresponding Bernoulli equation is:

$$\frac{P_\infty - P}{\rho} = \left\{[U-ry]\frac{\partial}{\partial x}-\cancelto{\sim 0}{pz\frac{\partial}{\partial y}}-\cancelto{\sim 0}{qx\frac{\partial}{\partial z}}+\frac{\partial}{\partial t}\right\}\Phi$$

$$-\frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right]\cong[U-ry]\frac{\partial\Phi}{\partial x}+\frac{\partial\Phi}{\partial t} \tag{4}$$

The crossed-out terms can be neglected, since the velocity perturbations are small compared to the freestream velocity $[U(t)\gg\partial\Phi/\partial x,\ \partial\Phi/\partial y,\ \partial\Phi/\partial z]$. Furthermore, in Eq. (4) the terms containing the angular velocities also can be neglected, as their contribution to the pressure difference across the wing is symmetrical. The general solution of Eq. (1) is a sum of doublet and source distributions. However, as wing thickness is not investigated here, only the asymmetric terms are considered for the present thin wing problem. As a result of this assumption, the solution to be chosen is a doublet σ distribution over the wing surface and the wake. Therefore, it is convenient to divide the velocity potential Φ into two parts:

$$\Phi = \Phi_f + \Phi_w \tag{5}$$

where Φ_f is the potential due to discontinuity distributions over the wing surface and Φ_w is the wake potential consisting of vortex elements shed from the trailing edge at previous time steps.

In order to solve the potential Φ, its derivative with respect to z (while $z\to 0$) is compared to the downwash in boundary

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condition, Eq. (2):

$$w(x,y,o)=\frac{\partial\Phi_f}{\partial z}+\frac{\partial\Phi_w}{\partial z}=\frac{1}{4\pi}\iint_{\text{wing}}\frac{\gamma(x_1y_1)}{(y-y_1)^2}$$

$$\times\left[1+\frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}}\right]\mathrm{d}x_1\mathrm{d}y_1+\Delta W_w$$

$$=(U-ry)\frac{\partial h}{\partial x}-qx+py+\frac{\partial h}{\partial t}\tag{6}$$

where

$$u^+-u^-=\gamma(xy)=\partial\sigma(xy)/\partial x\tag{7}$$

This integral equation should be evaluated in principal value terms and solved numerically for each time step. The wake influence ΔW_w is calculated by summing the downwash velocity of the wake vortex elements. When using the doublet distribution method, a straight, semi-infinite vortex wake is assumed, for which ΔW_w denotes the difference in the downwash caused by the nonsteady wake model used.

In order to deal with the arbitrary nonsteady motion of a wing and to enable the study of wake distortion, a detailed wake model was constructed.[1] The wake and wing circulations must fulfill Kelvin's law for each time step; i.e., the overall circulation generated in the flow must be zero, otherwise, the model will consist of closed vortex rings.

At each time step, when the solution of the wing circulation (or potential) is solved, the wake elements are allowed to deform in the wing-wake-induced velocity field. As a result of similar calculations, various wake patterns can be investigated[2,3] for various motions.

As previously stated, the application of steady wing theory solutions to the nonsteady flight problems in curved flight involves three major modifications. The first two are the wing surface downwash correction and a more detailed wake model, while the third change results from the additional $\partial\Phi/\partial t$ term in Eq. (4). The time derivative of the velocity potential can be obtained by evaluating the potential directly by integrating Eq. (6) or by using Eq. (8) to calculate the pressure difference due to $\partial(\Delta\Phi)/\partial t$:

$$\frac{\partial}{\partial t}[\Phi(x,y,0^+)-\Phi(x,y,0^-)]=\frac{\partial}{\partial t}\Delta\Phi(x,y)$$

$$=\frac{\partial}{\partial t}\int_{-\infty}^{x}[u^+(x_1,y_1,0^+)-u^-(x_1,y_1,0^-)]\mathrm{d}x_1$$

$$=\frac{\partial}{\partial t}\int_{\substack{x(y)\\ LE}}^{x}\gamma(x_1,y)\mathrm{d}x_1\tag{8}$$

Nonsteady Motion of a Slender-Thin Wing

As an example, the maneuvers of a slender thin wing are examined here. The wing moves along a given path; in the noninertial coordinate system, the momentary downwash $w(x,y,0)$ is given by Eq. (2). Because of the slenderness of the wing ($|z|,||x-x_1|\gg|y-y_1|$), the following reduction in the kernel function of Eq. (6) is obtained:

$$\left[1+\frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}}\right)\cong\begin{array}{l}2\text{ for }x_1<x\\0\text{ for }x_1>x\end{array}\tag{9}$$

Substituting Eq. (9) into Eq. (6) yields:

$$w(x,y,0)=\frac{1}{2\pi}\int_0^x\int_{-b(x)}^{+b(x)}\frac{\gamma(x_1y_1)}{(y-y_1)^2}\mathrm{d}x_1\mathrm{d}y_1+\Delta W_w$$

$$=-\frac{1}{2\pi}\int_0^x\int_{-b(x)}^{+b(x)}\frac{\partial\gamma}{\partial y_1}\frac{\mathrm{d}y_1}{(y-y_1)}\mathrm{d}x_1+\Delta W_w\tag{10}$$

The physical interpretation of Eq. (10) is that any spanwise section is influenced by the elements of the doublet sheet ahead of that section. However, in the nonsteady case when sudden accelerations are considered, the wake influence is included in the term ΔW_w of Eq. (6). Even so, the acceleration rates must be limited in such a way that the slender wing assumptions should hold, i.e., the wake-induced downwash on the wing should remain small $\{\partial\Delta W_w/\partial x_1\approx O[U(t)\alpha]\}$.

When the results of Eq. (8) are used to integrate Eq. (10) with x, the following relation is obtained:

$$w(x,y,0,t)=[U(t)-ry]\frac{\partial h}{\partial x}-qx+py+\frac{\partial h}{\partial t}$$

$$=-\frac{1}{2\pi}\int_{-b(x)}^{+b(x)}\frac{\partial\Delta\Phi(x,y_1)}{\partial y_1}\frac{\mathrm{d}y_1}{(y-y_1)}+\Delta W_w\tag{11}$$

The solution of Eq. (11) in terms of the spanwise coordinate λ is obtained by expanding Φ as

$$\Phi=\sum_{n=1}^{\infty}A_n(x,t)\sin n\lambda\tag{12}$$

where $y=b(x)\cos\lambda$, and coefficients A_n are calculated by integrating Eq. (11):

$$A_n\equiv A_n(x,t)=-\frac{2}{n\pi}\int_0^{\pi}W(x,y,t)b(x)\sin\lambda\,\sin n\lambda\,\mathrm{d}\lambda\tag{13}$$

and

$$W(x,y,t)\equiv[V(t)-ry]\frac{\partial h}{\partial x}-qx+py+\frac{\partial h}{\partial t}-\Delta W_w\tag{14}$$

The chordwise lift distribution $\mathrm{d}L/\mathrm{d}x$ is obtained with Eq. (4) by integrating the spanwise pressure distribution:

$$\frac{\mathrm{d}L}{\mathrm{d}x}=\int_{-b(x)}^{+b(x)}\Delta P\mathrm{d}y=2\rho\int_0^{2\pi}$$

$$\times\left\{\frac{\partial\Phi}{\partial t}+[V(t)-rb(x)\cos\lambda]\frac{\partial\Phi}{\partial x}\right\}b(x)\sin\lambda\mathrm{d}\lambda\tag{15}$$

while the derivatives, $\partial\Phi/\partial t$ and $\partial\Phi/\partial x$, are:

$$\frac{\partial\Phi}{\partial t}=\sum_{n=1}^{\infty}\frac{\partial A_n}{\partial t}\sin n\lambda\tag{16}$$

$$\frac{\partial\Phi}{\partial x}=\sum_{n=1}^{\infty}\frac{\partial A_n}{\partial x}\sin n\lambda+\frac{1}{b(x)tg\lambda}\frac{\mathrm{d}b(x)}{\mathrm{d}x}\sum_{n=1}^{\infty}A_n\cos n\lambda\tag{17}$$

Integration in Eq. (15) leads to:

$$\frac{\mathrm{d}L}{\mathrm{d}x}=\pi\rho\left\{b(x)\frac{\partial A_1}{\partial t}+U(t)\frac{\partial}{\partial x}[A_1b(x)]\right\}\tag{18}$$

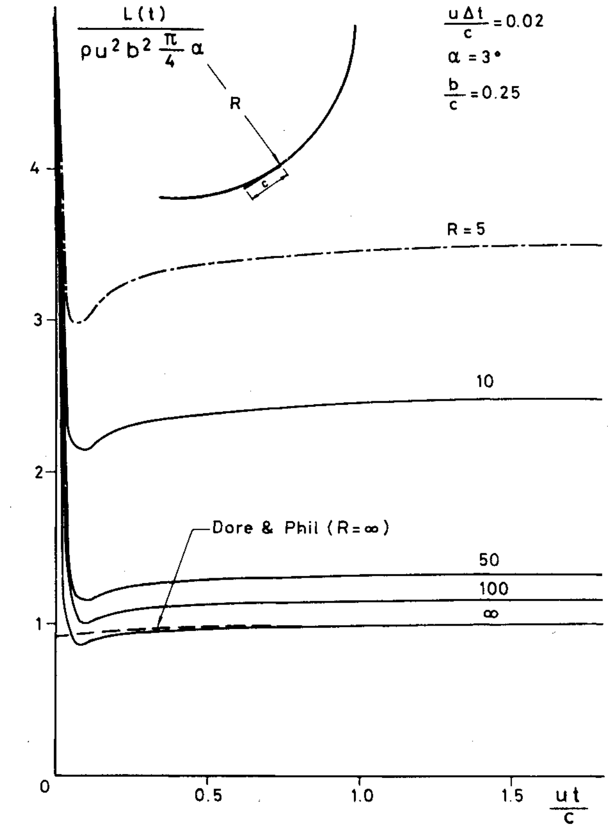

Fig. 1 Lift variation of a suddenly accelerated slender wing along a circular path.

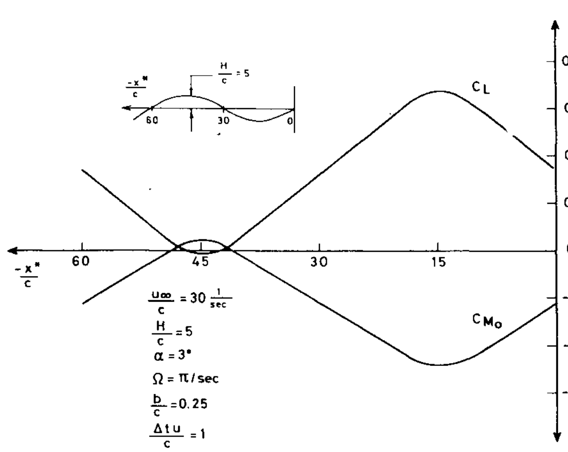

Fig. 2 Lift and moment variations along a sinusoidal path.

which can be reduced further to the known results for steady flight of a planar wing at incidence α:

$$\frac{dL}{dx} = \pi\rho U^2 \frac{db^2(x)}{dx} \alpha$$

The calculations of moments in the general case are carried out by numerical technique and are dependent on additional coefficients, such as A_2 for the rolling moment.

The time-dependent lift variations of a suddenly accelerated delta wing moving along a circular path are shown in Fig. 1. The very high forces at the beginning of the motion are due to the acceleration (added mass effect) of the surrounding fluid,

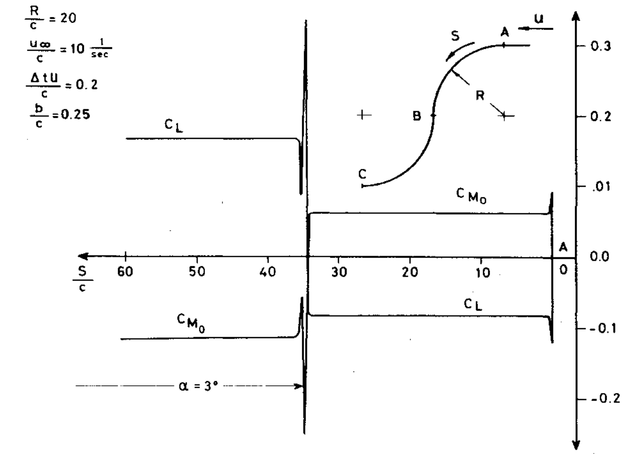

Fig. 3 Lift and moment variations during an S maneuver.

while the immediately following drop in lift is caused by the vortex wake shed during the acceleration period. For delta wings with small aspect ratio, the lift reaches its steady-state value in a short period. The dotted lines represent the results of Dore and Phil,[4] who calculated the sudden accelerations of various planforms along a straight path ($R = \infty$) via the indicial technique (without added mass effect). It is clear from Fig. 1 that smaller radius of curvature R causes higher lift values as a result of the upwash caused by the wing angular movement. However, for smaller path curvature ($R \leq 5$), the linear treatment underestimates the aerodynamic forces; therefore, no calculations were carried out for the region above the broken line of $R = 5$. The lift and moment variations along a sinusoidal path are presented in Fig. 2 (the moment is measured relative to wing leading edge). Here the wing is forced to follow the negative direction of an inertial x^* coordinate, while performing a sinusoidal motion with an amplitude of 5 c. Along this path, the angle of attack relative to the momentary flight direction is held constant at 3 deg. The considerable loss in the lift at the upper side of the path is due to the downwash caused by the angular movement of the wing.

Figure 3 shows the force variations along an S-shaped path. Here the surface approaches point A (Fig. 3) with a constant speed and zero angle of attack. At point A, it is forced to follow a curve with a radius R until point B. During the maneuver, the vehicle's potential energy is conserved ($U^2/2 + hg = \text{const}$) and, therefore, the flight velocity increases. At point B during a short period ($Ut/c = 0.2$), the path curvature is turned to the opposite side and the angle of attack is increased ($\alpha = 3$ deg). Figure 3, as well as Fig. 1, show that for a slender delta wing, the transient region is limited to the neighborhood of sudden changes (up to one chord length), while the magnitude of the forces and moment during the sudden variation in the wing's downwash depends mainly on the rate of this variation.

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